Math Behind DeFiHelper

DeFiHelper is a highly automated instrument for crypto asset investment management. With DHF, users can automate operations with their crypto assets, save a significant amount of time and generate more profits.

The DFH algorithm is able to shift from less profitable pools to more profitable ones without the intervention of the user. How does it calculate the optimal time for making a transfer? All that is needed is to have a clear understanding of the investor's income as a function on the basis of the following givens:

— The interest income accrued on the investment;

— The simple interest rate;

— The transaction fee.

The next step is to find some touch points at which the derivative of this function will be equal to zero or absent altogether, and give a recommendation to the investor to make the shift between instruments at appropriate time intervals. This step will allow the transformation of the simple interest into a complex one and result in the accrual of additional income, regardless of the fee charged for mining. Let us take a closer look at the process.

Some givens to begin with.

Let:

s — be the sum of the investment, $s > 0$;

p — be the accrued income, $p \geq 0$;

r — be the size of the simple interest rate, $r > 0$. Its size may vary:

in 1/year (for example, 50% annual equals $r = 0{,}5\ \frac{1}{\text{year}}$, 1% per day — equals $r = 3{,}65\ \frac{1}{\text{year}}$);

in 1/day (for example, 50% annual equals $r = \frac{0{,}5}{365}\ \frac{1}{\text{day}}$, 1% per day — equals $r = 0{,}01\ \frac{1}{\text{day}}$);

c — be the transaction fee, $c > 0$.

These values are constant.

Let us examine two types of strategies:

— The investor does not transfer the income to a deposit, but rather waits for time $t_1 + t_2$.

— The investor waits for time t_1, transfers the income to a deposit and pays a transaction fee c, and then waits for time t_2.

Let the following table illustrate all the sizes of all the assets (deposit amount + income that has not been transferred) for both strategies at different points in time:

Strategy	Time		
	0	t_1	$t_1 + t_2$
1	$s+p$	$s+p+s\cdot r\cdot t_1$	$s+p+s\cdot r\cdot t_1+s\cdot r\cdot t_2$
2	$s+p$	$s+p+s\cdot r\cdot t_1-c$	$s+p+s\cdot r\cdot t_1-c+(s+p+s\cdot r\cdot t_1-c)\cdot r\cdot t_2$

At the same time $t_1 > \frac{c-p}{s\cdot r}$, because otherwise, the income will not be large enough to pay the transaction fee.

Let the sizes of the assets for both strategies be equal at time $t_1 + t_2$. Then:

$$\begin{aligned}
&s+p+s\cdot r\cdot t_1+s\cdot r\cdot t_2 = s+p+s\cdot r\cdot t_1-c+(s+p+s\cdot r\cdot t_1-c)\cdot r\cdot t_2,\\
&s\cdot r\cdot t_2 = -c+(s+p+s\cdot r\cdot t_1-c)\cdot r\cdot t_2,\\
&s\cdot r\cdot t_2 = -c+s\cdot r\cdot t_2+p\cdot r\cdot t_2+s\cdot r^2\cdot t_1\cdot t_2-c\cdot r\cdot t_2,\\
&0=-c+p\cdot r\cdot t_2+s\cdot r^2\cdot t_1\cdot t_2-c\cdot r\cdot t_2,
\end{aligned} \tag{1}$$

where t_1 can be expressed as t_2 and vice versa:

$$t_1 = \frac{c+c\cdot r\cdot t_2-p\cdot r\cdot t_2}{s\cdot r^2\cdot t_2}, \tag{2.1}$$

$$t_2 = \frac{c}{s\cdot r^2\cdot t_1-c\cdot r+p\cdot r}. \tag{2.2}$$

Let us examine the following equation:

Find the minimal value of $t_1 + t_2$ (where $t_1 > \frac{c-p}{s \cdot r}$), or the minimal time at which the losses resulting from the payment of the transaction fee under strategy 2 will be covered by a larger size of the deposit.

If (2.2) is expressed as $t_1 + t_2$, then it is necessary to find the minimum value of the following equation (where $t_1 > \frac{c-p}{s \cdot r}$):

$$t_1 + t_2 = t_1 + \frac{c}{s \cdot r^2 \cdot t_1 - c \cdot r + p \cdot r} \tag{3}$$

Let us examine the following function $f(x)$:

$$f(x) = x + \frac{c}{s \cdot r^2 \cdot x - c \cdot r + p \cdot r}\,. \tag{4}$$

where $x > \frac{c-p}{s \cdot r}$, $c > 0$, $p \geq 0$, $s > 0$, $r > 0$.

Let us now try to find the minimum value of this function. In order to find the minimum of a function with one variable, as is the case here, we need to find the points at which the derivative of the function is equal to zero or does not exist at all. These will function as the extremes. We will then be able to compare the resulting value with the values of the derivative in the function under scrutiny. If the derivative changes from negative to positive when passing through the extreme value of the function, then we will have found its minimum value.

Let us find the derivative of this function for that end:

$$f'(x) = 1 - \frac{s \cdot r^2 \cdot c}{\left(s \cdot r^2 \cdot x - c \cdot r + p \cdot r\right)^2} = 1 - \frac{s \cdot c}{\left(s \cdot r \cdot x - c + p\right)^2}\,. \tag{5}$$

Now to find the points at which the derivative of the function vanishes or does not exist at all.

$$f'(x)=0 \Leftrightarrow 1-\frac{s\cdot c}{(s\cdot r\cdot x-c+p)^2}=0 \Leftrightarrow \frac{(s\cdot r\cdot x-c+p)^2-s\cdot c}{(s\cdot r\cdot x-c+p)^2}=0 \Leftrightarrow$$
$$\Leftrightarrow \begin{cases}(s\cdot r\cdot x-c+p)^2-s\cdot c=0,\\ (s\cdot r\cdot x-c+p)^2\neq 0,\end{cases} \Leftrightarrow \left[x>\frac{c-p}{s\cdot r} \Leftrightarrow s\cdot r\cdot x-c+p>0\right] \Leftrightarrow \tag{6}$$
$$\Leftrightarrow (s\cdot r\cdot x-c+p)^2-s\cdot c=0 \Leftrightarrow (s\cdot r\cdot x-c+p)^2=s\cdot c \Leftrightarrow$$
$$\Leftrightarrow \left[x>\frac{c-p}{s\cdot r} \Leftrightarrow s\cdot r\cdot x-c+p>0\right] \Leftrightarrow s\cdot r\cdot x-c+p=\sqrt{s\cdot c} \Leftrightarrow x=\frac{c-p+\sqrt{s\cdot c}}{s\cdot r}.$$

Point $x=\frac{c-p+\sqrt{s\cdot c}}{s\cdot r}$ is the minimum value of function $f(x)$, $f'(x)<0$ is on the left of the function, and $f'(x)>0$ is on the right.

Thus we obtain the minimum value for t_1+t_2 and the corresponding values for t_1 and t_2:

$$t_1=\frac{c-p+\sqrt{s\cdot c}}{s\cdot r}, \tag{7.1}$$

$$\begin{aligned}t_2 &= [see.\ (2.2)] = \frac{c}{s\cdot r^2\cdot t_1-c\cdot r+p\cdot r}\\ &= \frac{c}{s\cdot r^2\cdot\frac{c-p+\sqrt{s\cdot c}}{s\cdot r}-c\cdot r+p\cdot r} =\\ &= \frac{c}{r\cdot\left(c-p+\sqrt{s\cdot c}\right)-c\cdot r+p\cdot r}\\ &= \frac{c}{r\cdot c-r\cdot p+r\cdot\sqrt{s\cdot c}-c\cdot r+p\cdot r} =\\ &= \frac{c}{r\cdot\sqrt{s\cdot c}} = \frac{\sqrt{s\cdot c}}{s\cdot r}.\end{aligned} \tag{7.2}$$

$$\begin{aligned}t_1+t_2 &= [\text{see. } (7.1)\ and\ (7.2)] = \frac{c-p+\sqrt{s\cdot c}}{s\cdot r}+\frac{\sqrt{s\cdot c}}{s\cdot r}\\ &= \frac{c-p+2\sqrt{s\cdot c}}{s\cdot r}.\end{aligned} \tag{7.3}$$

Daily transfers

It would also be interesting to consider the formulas for calculating the sizes of the assets under a scenario involving daily transfers.

Let s_0 be the initial sum.

Let s_1 be the size of the deposit after 1 transfer:

$$s_1 = s_0 + p + s_0 \cdot r - c = s_0 \cdot (1+r) + p - c\ . \tag{8}$$

If we start repeating the same procedure with the same values of all the givens, then the size of the deposit after $2, 3, 4, ..., n$ transfers will be equal to:

$$s_2 = s_1 \cdot (1+r) - c = s_0 \cdot (1+r)^2 + p \cdot (1+r) - c \cdot (1+r) - c\ ,$$

$$s_3 = s_2 \cdot (1+r) - c = s_0 \cdot (1+r)^3 + p \cdot (1+r)^2 - c \cdot (1+r)^2 - c \cdot (1+r) - c\ ,$$

$$s_4 = s_3 \cdot (1+r) - c = s_0 \cdot (1+r)^4 + p \cdot (1+r)^3 - c \cdot (1+r)^3 - c \cdot (1+r)^2 - c \cdot (1+r) - c \tag{9}$$

,

$$\dots$$

$$s_n = ... = s_0 \cdot (1+r)^n + p \cdot (1+r)^{n-1} - c \cdot (1+r)^{n-1} - ... - c \cdot (1+r)^2 - c \cdot (1+r) - c\ ,$$

Which means that:

$$s_n = s_0 \cdot (1+r)^n + p \cdot (1+r)^{n-1} - c \cdot \frac{(1+r)^n - 1}{(1+r) - 1}\ . \tag{10}$$

As we can see, the scenario involving daily transfers does not differ too much from the general scenario and the accrual of additional income is the result of a combination of factors, such as the profitability of the new contract and the fee for the transfer, rather than the frequency of transfers.